

Unit 4: Logarithms ~ Learning Guide

Name: _____

Instructions:

Using a pencil, complete the following questions as you work through the related lessons.
Show ALL work as it is explained in the lessons. Do your best and ask your instructor if you don't understand any questions!

Lesson One

The Exponent Laws

1. Simplify the following and write answers with positive exponents only.

$$a) \frac{(x^2)^4}{x^{-3}}$$

$$\frac{x^8}{x^{-3}}$$

$$8+3 = 11$$

$$x^{11}$$

$$b) \frac{(m^2n^3)^{-1}}{m^3n}$$

$$\frac{1}{m^2n^3}$$

$$\frac{1}{m^2m^3}$$

$$\Rightarrow \frac{1}{m^5n^3}$$

$$c) \frac{(2x)^{-2}}{(3x^2)^3}$$

$$\frac{1}{(2x)^2}$$

$$\frac{1}{3^3x^6}$$

$$= \frac{1}{108x^6}$$

$$d) \left(\frac{x^2}{y}\right)^2 \left(\frac{2y^3}{x}\right)^{-1}$$

$$\frac{x^4}{y^2} \times \frac{1}{\frac{2y^3}{x}}$$

$$\frac{x^5}{2y^5}$$

$$e) \left(\frac{9y^6}{4x^2}\right)^{1/2} \div xy$$

$$\left(\frac{9y^6}{4x^2}\right)^{1/2}$$

$$\frac{3y^3}{2x}$$

$$\Rightarrow \frac{3y^3}{2x^2}$$

$$f) \frac{(3m)^{-1}}{(2n)^2} \left(\frac{n}{m^2}\right)^3$$

$$\frac{1}{4m^2} \times \frac{n^3}{m^6}$$

$$\frac{n^3}{12m^8}$$

$$g) (xy)^2 \left(\frac{1}{2x^3}\right)^{-2} \div y$$

$$\frac{x^2y^2 \times 4x^6}{y}$$

$$4x^8y$$

$$h) \left(\frac{(2m^2)^{-1}}{n^3}\right)^2 \div \left(\frac{m}{3n^3}\right)^4$$

$$\frac{\left(\frac{1}{2m^2}\right)^2}{n^6}$$

$$\frac{\frac{1}{4m^4}}{n^6}$$

$$= \frac{1}{4m^4n^6}$$

$$= \frac{81m^6}{4n^8}$$

2. Evaluate the expressions for $x = -1$, $y = 2$, $z = 3$.

$$\begin{aligned} \text{a) } (3x^2y^{-1}z^2)(4x^{-1}y^2z) \\ \frac{3 \times (-1)^2 \times (3)^2}{2} \times \frac{4 \times (-1)^2 \times 3}{-1} \\ = -648 \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{(2xy^3z^{-1})^2}{x^2y^4} \\ \frac{\left(\frac{2 \times (-1) \times (2)^3}{3}\right)^2}{(-1)^2 \times (2)^4} = \frac{16}{9} \end{aligned}$$

Lesson Two

Solving Equations Involving Exponents

1. Solve for x .

$$\begin{aligned} \text{a) } x^{2/3} &= 4 \\ x^2 &= (4)^3 \\ x^2 &= 64 \\ x &= 8 \end{aligned}$$

$$\begin{aligned} \text{b) } x^{-1/2} &= 9 \\ \frac{1}{x^{1/2}} &= 9^2 \\ x &= \frac{1}{81} \end{aligned}$$

$$\begin{aligned} \text{c) } 5x^{3/2} &= 40 \\ x^{3/2} &= \frac{40}{5} \\ x^{3/2} &= 8^2 \\ x &= 4 \end{aligned}$$

$$\begin{aligned} \text{d) } (4x+1)^{2/3} &= 25^{3/2} \\ (4x+1) &= (25)^{3/2} \\ 4x+1 &= 125 \\ 4x &= 124 \\ x &= 31 \end{aligned}$$

2. Solve for x by converting to common bases, show all your work.

$$\begin{aligned} \text{a) } 2^x &= 32 \\ 2^x &= 2^5 \\ x &= 5 \end{aligned}$$

$$\begin{aligned} \text{b) } \left(\frac{1}{3}\right)^x &= 81 \\ \left(\frac{1}{3}\right)^x &= 3^4 \\ 3^{-x} &= 3^4 \\ x &= -4 \end{aligned}$$

$$\begin{aligned} \text{c) } 5^x &= 625 \\ 5^x &= 5^4 \\ x &= 4 \end{aligned}$$

$$\begin{aligned} \text{d) } 3^{x-2} &= 27 \\ 3^{x-2} &= 3^3 \\ x-2 &= 3 \\ x &= 5 \end{aligned}$$

$$\begin{aligned} \text{e) } 4^{2x} &= 8 \\ 4^{2x} &= 2^3 \\ 2^{4x} &= 2^3 \\ 4x &= 3 \\ x &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{f) } 27^{x-1} &= 9 \\ 3^{3(x-1)} &= 3^2 \\ 3(x-1) &= 2 \\ x-1 &= \frac{2}{3} \\ x &= \frac{5}{3} \end{aligned}$$

$$\begin{aligned} \text{g) } 3 \cdot 16^{x+2} &= 96 \\ 16^{x+2} &= 32 \\ 4^{x+2} &= 5 \\ 2 &= 2 \\ 4(x+2) &= 5 \\ x+2 &= \frac{5}{4} \\ x &= -\frac{3}{4} \end{aligned}$$

$$\begin{aligned} \text{h) } 25^{2x-1} &= 5^x \\ 5^{2(2x-1)} &= 5^x \\ 2(2x-1) &= x \\ 4x-2 &= x \\ 3x &= 2 \\ x &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{i) } 2 \cdot 8^{x+1} &= 16 \\ 8^{x+1} &= 8 \\ x+1 &= 1 \\ x &= 0 \end{aligned}$$

$$\begin{aligned} \text{j) } 4^x &= 8^{x+1} \\ 2^{2x} &= 2^{3(x+1)} \\ 2 &= 2 \\ 2x &= 3x+3 \\ x &= -3 \end{aligned}$$

$$\begin{aligned} \text{k) } 3^{3x+4} &= 9^x \\ 3^{3x+4} &= 3^{2x} \\ 3 &= 3 \\ 3x+4 &= 2x \\ x &= -4 \end{aligned}$$

$$\begin{aligned} \text{l) } \left(\frac{1}{4}\right)^{-x+2} &= 8^{3x-1} \\ 4^{x-2} &= 8^{3x-1} \\ 2^{2(x-2)} &= 2^{3(3x-1)} \\ 2 &= 2 \\ 2(x-2) &= 3(3x-1) \\ 2x-4 &= 9x-3 \\ x &= -\frac{1}{7} \end{aligned}$$

Lesson Three

Defining a Logarithm and Logarithmic Restrictions

1. Convert the following exponential equations to logarithmic form.

$$\begin{array}{llll} \text{a) } 4^0 = 1 & \text{b) } 5^2 = 25 & \text{c) } 9^{1/2} = 3 & \text{d) } a^b = c \\ \log_4 1 = 0 & \log_5 25 = 2 & \log_9 3 = \frac{1}{2} & \log_a c = b \end{array}$$

2. Convert the following logarithmic equations to exponential form.

$$\begin{array}{llll} \text{a) } \log_3 9 = 2 & \text{b) } \log_4 \frac{1}{16} = -2 & \text{c) } \log 0.1 = -1 & \text{d) } \log_a b = c \\ 3^2 = 9 & 4^{-2} = \frac{1}{16} & 10^{-1} = 0.1 & a^c = b \end{array}$$

3. Evaluate each of the following.

a) $\log_{10} 1000$

$$\log_{10} 10^3 = 3$$

b) $\log_2 16$

$$\log_2 2^4 = 4$$

c) $\log_9 \frac{1}{3}$

$$\log_9 \left(\frac{1}{9}\right)^{\frac{1}{2}} = -\frac{1}{2}$$

d) $\log_2 8\sqrt{2}$

$$\log_2 2^3 \times 2^{\frac{1}{2}} = \log_2 2^{\frac{7}{2}} = \frac{7}{2}$$

4. Solve for x.

a) $\log_4 64 = x$

$$\log_4 4^3 = x \\ 3 = x$$

b) $\log_3 9 = 2$

$$x^2 = 9 \\ x = 3$$

c) $\log_5 x = 3$

$$5^3 = x \\ x = 125$$

d) $\log_3 27 = \frac{3}{2}$

$$x^{\frac{3}{2}} = 27 \\ x = 27^{\frac{2}{3}} \\ x = 9$$

5. State the domain for each of the following.

a) $y = \log_3 (x+5)$

$$x+5 > 0$$

Domain $x > -5$

b) $y = \log_2 (2x-3)$

$$2x-3 > 0$$

Domain $x > \frac{3}{2}$

c) $y = \log_4 (3-x)$

$$3-x > 0 ; x > 0 ; x \neq 1$$

$$x < 3 ; x > 0 ; x \neq 1$$

Domain $0 < x < 3 ; x \neq 1$

Lesson FourLaws of Logarithms

1. Write the following as a single logarithm.

a) $\log x + \log y$

$$\log xy$$

b) $2\log_3 x - \log_3 y$

$$\log_3 x^2 - \log_3 y$$

$$\log_3 \frac{x^2}{y}$$

c) $\log x + 5\log y - 3\log x$

$$\log x + \log y^5 - \log x^3$$

$$\log \frac{xy^5}{x^3} = \log \frac{y^5}{x^2}$$

d) $-\log x + \frac{1}{2}\log y + 1$

$$\log \frac{1}{x} + \log \sqrt{y} + \log 10$$

$$\log \frac{10\sqrt{y}}{x}$$

e) $\frac{1}{4}\log_2 x + 3\log_2 y - 3$

$$\log_2 x^{\frac{1}{4}} + \log_2 y^3 - \log_2 (2)^3$$

$$\log_2 \frac{x^{\frac{1}{4}} y^3}{8}$$

2. Expand.

a) $\log \frac{x}{y^2}$

$$\log x - \log y^2$$

$$\log x - 2\log y$$

b) $\log_2 (\sqrt{xy})$

$$\log_2 x^{\frac{1}{2}} + \log_2 y$$

$$\frac{1}{2}\log_2 x + \log_2 y$$

c) $\log \left(\frac{y}{x}\right)^2$

$$\log y^2 - \log x^2$$

$$2\log y - 2\log x$$

d) $\log \frac{100x}{z^2 \sqrt[5]{y}}$

$$\log 100 + \log x - \log z^2 - \log y^{\frac{1}{5}}$$

$$2 + \log x - 2\log z - \frac{1}{5}\log y$$

e) $\log_2 \frac{\sqrt[3]{x^2 y}}{16}$

$$\log_2 x^{\frac{2}{3}} + \log_2 y^{\frac{1}{3}} - \log_2 16$$

$$\frac{2}{3}\log_2 x + \frac{1}{3}\log_2 y - 4$$

3. Given that $\log_a x = 2$ and that $\log_a y = 3$ then evaluate the following.

$$\begin{aligned} \text{a) } \log_a x^2 y \\ \log_a x^2 + \log_a y \\ 2 \log_a x + \log_a y \\ = 7 \end{aligned}$$

$$\begin{aligned} \text{b) } \log_a \left(\frac{ay}{x} \right) \\ \log_a a + \log_a y - \log_a x \\ 1 + \log_a y - \log_a x \\ 2 \end{aligned}$$

$$\begin{aligned} \text{c) } 3 \log_a (\sqrt{xy}) \\ 3 \left[\log_a x^{\frac{1}{2}} + \log_a y \right] \\ 3 \left[\frac{1}{2} \log_a x + \log_a y \right] \\ = 3 \times 4 \\ = 12 \end{aligned}$$

4. Simplify each of the following.

$$\begin{aligned} \text{a) } 5^{2 \log_5 x} \\ 5^{\log_5 x^2} \\ = x^2 \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{1}{\log_3 2} + \frac{1}{\log_5 2} \\ \frac{\log_2 3}{\log_2 2} + \frac{\log_2 5}{\log_2 2} \\ \log_2 3 + \log_2 5 \\ \log_2 15 = \log_2 15 \end{aligned}$$

$$\begin{aligned} \text{c) } 8^{\log_2 x} \\ 2^{3 \log_2 x} \\ 2^{\log_2 x^3} \\ = x^3 \end{aligned}$$

5. Given that $\log 2 = a$ and that $\log 3 = b$ then determine an expression for the following in terms of a and b without using a calculator.

$$\begin{aligned} \text{a) } \log 6 \\ \log(3 \times 2) \\ \log 3 + \log 2 \\ a + b \end{aligned}$$

$$\begin{aligned} \text{b) } \log \left(\frac{30}{16} \right) \\ \log \left(\frac{3 \times 10}{2^4} \right) \\ \log 10 + \log 3 - 4 \log 2 \\ 1 + b - 4a \end{aligned}$$

$$\begin{aligned} \text{c) } \log 1200 \\ \log(100 \times 3 \times 2^2) \\ \log 100 + \log 3 + 2 \log 2 \\ 2 + b + 2a \end{aligned}$$

6. If $\log_3 x = 10$ then evaluate each of the following without a calculator.

$$\begin{aligned} \text{a) } \log_3 (9x) \\ \log_3 (3^2) + \log_3 x \\ = 2 + 10 \\ = 12 \end{aligned}$$

$$\begin{aligned} \text{b) } \log_3 \left(\frac{27}{x} \right) \\ \log_3 (3^3) - \log_3 x \\ = 3 - 10 \\ = -7 \end{aligned}$$

$$\begin{aligned} \text{c) } \log_3 \left(\frac{x^2}{3} \right) \\ 2 \log_3 x - \log_3 3 \\ = 20 - 1 \\ = 19 \end{aligned}$$

7. Given that $\log_3 4 = a$ and $\log_3 5 = b$, find an expression for $\log_3 20$ in terms of a and b without using a calculator.

$$\begin{aligned}\log_3 4 &= a & \log_3 5 &= b \\ \frac{\log 4}{\log 3} &= a & \frac{\log 5}{\log 3} &= b \\ \frac{1}{2} \log_3 4 &= a \Rightarrow \log_3 4 = 2a & \log_3 5 &= 4b\end{aligned}$$

$$\begin{aligned}\log_3 20 &= \log_3 4 \times 5 \\ &= \log_3 4 + \log_3 5 \\ &= 2a + 4b\end{aligned}$$

Lesson Five

Solving Exponential and Logarithmic Equations & Introduction to Exponential Functions

1. Solve the following exponential equations for the exact value of x then evaluate your solution rounding to 4 decimal places.

a) $3^x = 11$

$$\log 3^x = \log 11$$

$$\begin{aligned}x &= \frac{\log 11}{\log 3} \\ &= 2.1827\end{aligned}$$

b) $5^{x-1} = 7$

$$\log 5^{x-1} = \log 7$$

$$(x-1) = \frac{\log 7}{\log 5}$$

$$x = \frac{\log 7}{\log 5} + 1 = 2.2091$$

c) $7^{3x} + 3 = 21$

$$7^{3x} = 18$$

$$\log 7^{3x} = \log 18$$

$$3x = \frac{\log 18}{\log 7}$$

$$x = \frac{1}{3} \frac{\log 18}{\log 7}$$

$$x = 0.4951$$

d) $1 = 2 \cdot 4^x - 5$

$$6 = 2 \times 4^x$$

$$4^x = 3$$

$$\log 4^x = \log 3$$

$$x = \frac{\log 3}{\log 4} = 0.7213$$

$$c) 3^{n+1} = 5^n$$

$$\log 3^{n+1} = \log 5^n$$

$$(n+1) \log 3 = n \log 5$$

$$\frac{n+1}{n} = \frac{\log 5}{\log 3}$$

$$1 + \frac{1}{n} = \frac{\log 5}{\log 3}$$

$$\frac{1}{n} = \frac{\log 5}{\log 3} - 1$$

$$n = \frac{\log 3}{\log 5 - \log 3} = 2.1507$$

$$d) 5 \cdot 2^{3n-2} = 9^n$$

$$\log 5 + \log 2^{3n-2} = \log 9^n$$

$$\log 5 + (3n-2) \log 2 = n \log 9$$

$$\log 5 + 3n \log 2 - 2 \log 2 = n \log 9$$

$$3n \log 2 - n \log 9 = 2 \log 2 - \log 5$$

$$n [3 \log 2 - \log 9] = 2 \log 2 - \log 5$$

$$n = \frac{2 \log 2 - \log 5}{3 \log 2 - \log 9} = 1.8945$$

$$g) 6^{2n+1} = 7^{1-n}$$

$$\log 6^{2n+1} = \log 7^{1-n}$$

$$(2n+1) \log 6 = (1-n) \log 7$$

$$2n \log 6 + \log 6 = \log 7 - n \log 7$$

$$2n \log 6 + n \log 7 = \log 7 - \log 6$$

$$n [2 \log 6 + \log 7] = \log 7 - \log 6$$

$$n = \frac{\log 7 - \log 6}{2 \log 6 + \log 7} = 0.0279$$

2. Solve the following algebraically for x.

a) $\log_2 x = 0$

$$2^0 = x$$

$$x = 1$$

b) $\log(x-2) = 1$

$$10^1 = x-2$$

$$10 = x-2$$

$$x = 12$$

c) $\log 15 - \log 5 = \log x$

$$\log \frac{15}{5} = \log x$$

$$\log 3 = \log x$$

$$x = 3$$

d) $\log_3(x-2) - \log_3 2 = 3$

$$\log_3 \left(\frac{x-2}{2} \right) = 3$$

$$\frac{3}{3} = \frac{x-2}{2}$$

$$27 = \frac{x-2}{2} \Rightarrow x-2 = 54$$

$$x = 56$$

e) $\log x + \log x = \log 36$

$$\log x^2 = \log 36$$

$$x^2 = 36$$

$$x = 6$$

f) $2\log_3(1-x) = 6$

$$\log_3(1-x) = \frac{6}{2} = 3$$

$$3^3 = 1-x$$

$$x = -26$$

g) $\log(x-1) + \log(x+2) = 1$

$$\log(x-1)(x+2) = 1$$

$$(x-1)(x+2) = 10$$

$$x^2 + x - 2 = 10$$

$$x^2 + x - 12 = 0$$

$$x^2 + 4x - 3x - 12 = 0$$

$$x(x+4) - 3(x+4) = 0$$

$$(x-3)(x+4)$$

$$x = 3$$

$$x \neq -4 \quad \text{Not in Domain}$$

h) $\log(2x+1) = 1 + \log(1-x)$

$$\log(2x+1) - \log(1-x) = \log 10$$

$$\frac{(2x+1)}{1-x} = 10$$

$$2x+1 = 10-10x$$

$$12x = 9$$

$$x = \frac{9}{12} = \frac{3}{4}$$

$$i) \log_2(x^2 - 8) = 3$$

$$x^2 - 8 = 2^3$$

$$x^2 - 8 = 8$$

$$x^2 = 16$$

$$x = \pm 4$$

$$j) \log_2(x+5) = 1 - \log_2(x+3)$$

$$\log_2(x+5) + \log_2(x+3) = 1$$

$$(x+5)(x+3) = 2^1$$

$$x^2 + 8x + 15 = 2$$

$$x^2 + 8x + 13 = 0$$

$$x^2 + 6x + 2x + 12 = 0$$

$$x(x+6) + 2(x+6) = 0$$

$$(x+2)(x+6) = 0$$

$$x = -2$$

$$x \neq -6 \text{ Not in Domain}$$

$$k) \log_2(x-3) - \log_2 5 + \log_2(x-2) = 2$$

$$\log_2 \left[\frac{(x-3)(x-2)}{5} \right] = 2$$

$$\frac{(x-3)(x-2)}{5} = 2^2$$

$$(x-3)(x-2) = 20$$

$$x^2 - 5x + 6 = 20$$

$$x^2 - 5x - 14 = 0$$

$$x^2 - 7x + 2x - 14 = 0$$

$$x(x-7) + 2(x-7) = 0$$

$$(x+2)(x-7) = 0$$

$$x = 7$$

$$x \neq -2 \text{ Not in Domain}$$

$$l) \log_2(\log_5 25) = x$$

$$\frac{x}{2} = \log_5 25$$

$$2^{\frac{x}{2}} = \log_5 5^2$$

$$2^{\frac{x}{2}} = 2$$

$$x = 1$$

Lesson Six, Seven and EightApplications of Exponential Functions

1. Sarah invests \$1200 at a rate of 5% per year compounded monthly.

a) How much money would Sarah have after 7 years?

$P = 1200$

$R = 5\%$

$t = 7 \text{ years}$

$m = \text{Compounded Monthly} = 12$

$$A = P \left(1 + \frac{R}{100 \times m} \right)^{t \times m}$$

$$= 1200 \left[1 + \frac{5}{12 \times 100} \right]^{7 \times 12} = 1701.64$$

b) How long does it take for the investment to grow to \$2000?

$A = 2000$

$P = 1200$

$R = 5$

$t = ?$

$m = \text{Compounded Monthly} = 12$

$$2000 = 1200 \left[1 + \frac{5}{12 \times 100} \right]^{t \times 12}$$

$$\frac{2000}{1200} = \left[1 + \frac{5}{1200} \right]^{12t}$$

$$\ln \left[\frac{20}{12} \right] = 12t \ln \left[1 + \frac{5}{1200} \right]$$

$$t = \frac{\ln \left[\frac{20}{12} \right]}{12 \ln \left[\frac{1205}{1200} \right]} = 10.24 \text{ years}$$

2. An investment of \$3000 lost 4% per year compounded weekly.

a) How much is the investment worth after 5 years?

$P = 3000$

$R = -4\%$

$t = 5 \text{ years}$

$\text{Compounded weekly } m = 52$

$$A = 3000 \left[1 - \frac{4}{52 \times 100} \right]^{5 \times 52}$$

$$= 2456$$

b) How long does it take for the investment to be reduced to half of the initial amount?

$P = 3000$

$A = 1500$

$t = ?$

$R = -4\%$

$\text{Compounded weekly } m = 52$

$$1500 = 3000 \left[1 - \frac{4}{52 \times 100} \right]^{t \times 52}$$

$$\frac{1}{2} = \left[1 - \frac{4}{5200} \right]^{52t}$$

$$\ln \frac{1}{2} = 52t \ln \left[1 - \frac{4}{5200} \right]$$

$$t = \frac{\ln \frac{1}{2}}{52 \ln \left[1 - \frac{4}{5200} \right]} = 17.32 \text{ years}$$

3. At what rate of interest would an investment have to be paid for an initial \$900 to grow into \$1400 in 12 years if the investment was compounded quarterly?

$$A = 1400$$

$$P = 900$$

$$R = ?$$

$$t = 12 \text{ yr}$$

$$n = 4$$

$$A = P \left(1 + \frac{r}{n \times 100} \right)^{n \times t}$$

$$\frac{A}{P} = \left[1 + \frac{r}{n \times 100} \right]^{n \times t}$$

$$\left[1 + \frac{r}{n \times 100} \right] = \left(\frac{A}{P} \right)^{\frac{1}{n \times t}}$$

$$1 + \frac{r}{n \times 100} = \left(\frac{A}{P} \right)^{\frac{1}{n \times t}}$$

$$\frac{14}{9} = \left[1 + \frac{r}{4 \times 100} \right]^{\frac{1}{12 \times 4}}$$

$$\frac{14}{9} = \left[1 + \frac{r}{400} \right]^{\frac{1}{12}}$$

$$\left[1 + \frac{r}{400} \right] = \left(\frac{14}{9} \right)^{12}$$

$$1 + \frac{r}{400} = 2.17$$

4. A population of 1200 badgers grows by 6% every 8 years. How long does it take the population of badgers to grow to 2000?

$$2000 = 1200 \left[1 + \frac{6}{100} \right]^{\frac{t}{8}}$$

$$(1.06)^{\frac{t}{8}} = 1.66$$

$$\frac{t}{8} = \frac{\log 1.66}{\log 1.06}$$

$$t = 70.13 \text{ years}$$

5. The intensity of light decreases by 4% for each meter that it descends below the surface of the water. At what depth is the intensity of light only 20% of that at the surface?

$$I = 1 \times \left[1 - \frac{4}{100}\right]^x$$

$$0.2 = 0.96^x$$

$$x = \frac{\log 0.2}{\log 0.96} = 39.43 \text{ meters}$$

6. A colony of bees doubles in population every 5 weeks. How long does it take for the population to triple?

Let the number of week be w

$$(2)^{\left(\frac{w}{5}\right)} = 3$$

$$\frac{w}{5} \log 2 = \log 3$$

$$w = 7.92 \text{ weeks}$$

7. The half-life of a certain radioactive isotope is 20 days. How long does it take for 50g of the isotope to decay to 10g?

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$10 = 50 \left(\frac{1}{2}\right)^{\frac{t}{20}}$$

$$\frac{1}{5} = \left(\frac{1}{2}\right)^{\frac{t}{20}}$$

$$\frac{t}{20} \log \frac{1}{2} = \log \frac{1}{5}$$

$$t = 20 \times \frac{\log(\frac{1}{5})}{\log(\frac{1}{2})} = 46.44 \text{ days}$$

8. A radioactive substance is produced from nuclear fallout. If 600g of the substance decays to 250g in 20 years what is its half-life?

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$250 = 600 \left(\frac{1}{2}\right)^{\frac{20}{t_{1/2}}}$$

$$\frac{250}{600} = \left(\frac{1}{2}\right)^{\frac{20}{t_{1/2}}}$$

$$\frac{20}{t_{1/2}} = \frac{\log\left(\frac{250}{600}\right)}{\log\left(\frac{1}{2}\right)}$$

$$t_{1/2} = \frac{20 \times \log\left(\frac{1}{2}\right)}{\log\left(\frac{250}{600}\right)} = 15.83 \text{ years}$$